

Introduction to the Gnu Linear Programming Kit

Optimizing financial and industry models with GLPK

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● Gnu Linear Programming Kit

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● Gnu Linear Programming Kit

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- organized as a callable library



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- organized as a callable library
- written in ANSI C



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- objectives
 - ◆ solve linear programming problems
 - ◆ solve mixed integer programming problems
 - ◆ solve some other related problems



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 - ◆ solve linear programming problems
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- current version: 4.20



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- objectives
 - ◆ solve linear programming problems
 - ◆ solve mixed integer programming problems
 - ◆ solve some other related problems
- current version: 4.20
- home page: <http://www.gnu.org/software/glpk/glpk.html>



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■ revised simplex algorithm



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 - ◆ glpsol
 - default solver in GLPK package



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- standalone solver
 - ◆ glpsol
 - default solver in GLPK package
- other options
 - ◆ glpkmex - the Matlab MEX Interface of GLPK
 - ◆ DELI - interface for Delphi
 - ◆ JNI interface



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- Free MPS format



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- CPLEX LP format



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- CPLEX LP format
- GNU MathProg modeling language



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- Free MPS format
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This tutorial focuses on the Gnu MathProg.



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From Winston, Wayne L. - Operations Research

- *two types of wooden toys: soldiers and trains*
 - *soldier: sells for £27, uses £10 worth of raw materials, increases variable labor and overhead costs by £14*
 - *train: sells for £21, uses £9 worth of raw materials, increases variable labor and overhead costs by £10*
 - *soldier: requires 2 hours of finishing labor and 1 hour of carpentry labor*
 - *train: requires 1 hour of finishing labor and 1 hour of carpentry labor*
 - *Maximum of 100 finishing hours and 80 carpentry hours is available weekly*
 - *Weekly demand: trains (unlimited), soldiers (40)*
- Giapetto wants to maximize weekly profits (revenues - costs)**



Two-var problem: Giapetto's Woodcarving

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■ Decision variables

x_1 : Soldiers produced each week

x_2 : Trains produced each week

■ Objective function

$$(1) \quad \max z = (27x_1 + 21x_2) - (10x_1 + 9x_2) - (14x_1 + 10x_2) = 3x_1 + 2x_2$$

■ Constraints

$$(2) \quad 2x_1 + x_2 \leq 100 \quad (\textit{finishing constraint})$$

$$(3) \quad x_1 + x_2 \leq 80 \quad (\textit{carpentry constraint})$$

$$(4) \quad x_1 \leq 40 \quad (\textit{constraint on demand for soldiers})$$

$$(5) \quad x_1 \geq 0, \quad x_2 \geq 0 \quad (\textit{sign constraints})$$



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 - ◆ St
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Enhancing the model

- Parameters
- Data section
- Summation



The diet problem

From Winston, Wayne L. - Operations Research

Satisfy my daily nutritional requirements at minimum cost

Type of food	Cost per unity
Brownie	£0.5
Chocolate ice cream	£0.2 (scoop)
Cole	£0.3 (bottle)
Pineapple cheesecake	£0.8 (piece)

Daily needs	Amount
Calories	500
Chocolate	6 oz
Sugar	10 oz
Fat	8 oz

Type of food	Calories	Chocolate(oz)	Sugar(oz)	Fat(oz)
Brownie	400	3	2	2
Chocolate ice cream (scoop)	200	2	2	4
Cole (1 bottle)	150	0	4	1
Pineapple cheesecake (1 piece)	500	0	4	5

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■ Decision variables

x_1 : number of brownies eaten daily

x_2 : number of scoops of chocolate ice cream eaten daily

x_3 : bottles of cola trunk daily

x_4 : pieces of pineapple cheesecake eaten daily

■ Objective function

$$(1) \quad \min z = 50x_1 + 20x_2 + 30x_3 + 80x_4$$

■ Constraints

$$(2) \quad 400x_1 + 200x_2 + 150x_3 + 500x_4 \geq 500 \quad (\text{Calorie constraint})$$

$$(3) \quad 3x_1 + 2x_2 \geq 6 \quad (\text{Chocolate constraint})$$

$$(4) \quad 2x_1 + 2x_2 + 4x_3 + 4x_4 \geq 10 \quad (\text{Sugar constraint})$$

$$(5) \quad 2x_1 + 4x_2 + x_3 + 5x_4 \geq 8 \quad (\text{Fat constraint})$$

$$(6) \quad x_i \geq 0, \quad \forall i \in \{1, \dots, 4\}$$



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- Enhancements
 - ◆ 2-dimensional table



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 - ◆ 2-dimensional table
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Modified from Winston, Wayne L. - Operations Research

- *Semicond manufactures tape recorders and radios*
- *costs, and selling price are given in one of the tables*
- *available raw material sufficient to manufacture 100 tape recorders and 100 radios*
- *balance sheet is shown in one of the tables*
- *asset-liability ratio is $20000/10000 = 2$*
- *Demand: unlimited*
- *Semicond will collect £2000 in accounts receivable*
- *Semicond must pay off £1000 of the outstanding loan and a monthly rent of £1000*
- *January 1, 2008: receive a shipment of raw material worth £2000*
- *cash balance must be at least £4000*
- *current ratio must be at least 2*

What should Semicond produce on December?



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Problem data

	Tape recorder	Radio
Selling price	£130	£110
Labor cost	£50	£35
Raw material cost	£30	£40

	Asserts	Liability
Cash	£10.000	
Accounts receivable	£3.000	
Inventory outstanding	£7.000	
Bank loan		£10.000



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x_1 : tapes produced

x_2 : radios produced

■ Objective function

$$(1) \quad \max z = 50x_1 + 35x_2$$

■ Constraints

$$(2) \quad x_1 \leq 100 \text{ (tape constraint)}$$

$$(3) \quad x_2 \leq 100 \text{ (radio constraint)}$$

$$(4) \quad 50x_1 + 35x_2 \leq 6000 \text{ (cash position constraint)}$$

$$(5) \quad 50x_1 + 35x_2 \geq 2000 \text{ (ratio constraint)}$$

$$(6) \quad x_1 \geq 0, \quad x_2 \geq 0$$



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- **Multiple solution problem!**



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Modified from Winston, Wayne L. - Operations Research

- *Rylon Corporation manufactures Brute and Chanelle perfumes*
- *raw material: purchased for £3 per pound*
- *processing 1lb of raw material: 1 hour of laboratory time*
- *each pound of raw-material: 3oz of Regular Brute Perfume and 4oz of Regular Chanelle Perfume*
- *Regular Brute sells for £7/oz and Regular Chanelle for £6/oz*
- *reprocessing: Luxury Brute, sold at £18/oz, and Luxury Chanelle, sold at £14/oz*
- *Each oz of Regular Brute processed further: additional of 3 hours of lab time and £4 processing cost, yields 1oz of Luxury Brute*
- *Each oz of regular Chanelle processed further: additional 2 hours of lab time and £4 processing cost, yields 1oz of Luxury Chanelle*
- *yearly: 6000 hours of lab time available and can purchase up to 4000lb of raw material*

Maximize Rylon's profit



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■ Decision variables

x_1 : No. of oz of Regular Brute sold annually

x_2 : No. of oz of Luxury Brute sold annually

x_3 : No. of oz of Regular Chanelle sold annually

x_4 : No. of oz of Luxury Chanelle sold annually

x_5 : No. of pounds of raw material purchased annually

■ Objective function

$$(1) \quad \max z = 7x_1 + 14x_2 + 6x_3 + 10x_4 - 3x_5$$



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■ Constraints

$$(1) \quad x_5 \leq 4000 \quad (\text{raw material})$$

$$(2) \quad 3x_2 + 2x_4 + x_5 \leq 6000 \quad (\text{lab hours})$$

$$(3) \quad x_1 + x_2 - 3x_5 = 0 \quad (\text{mass conservation})$$

$$(4) \quad x_3 + x_4 - 4x_5 = 0 \quad (\text{mass conservation})$$

$$(5) \quad x_i \geq 0, \quad \forall i \in \{1, \dots, 5\}$$



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- Let's modify it a little bit: no mass conservation
- What should the result be?



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- What should the result be?
- **Unbounded problem!**



Multi-period investments

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From Winston, Wayne L. - Operations Research

- *Finco needs an investment strategy for the next three years*
- *There are five investments available*
- *Cash flow for each invested £1 is in the table below*
- *At most £75000 should be placed in any investment*
- *Finco can earn 8% per year with money market funds with uninvested cash*
- *Finco can not borrow money*
- *Finco has £100000 available to invest*

Investment	Time 0	Time 1	Time 2	Time 3
A	-1	+0.5	+1	0
B	0	-1	+0.5	+1
C	-1	+1.2	0	0
D	-1	0	0	+1.9
E	0	0	-1	+1.5

Maximize Finco's cash by the end of the third year



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■ Decision variables

x_1 : pounds invested in investment A

x_2 : pounds invested in investment B

x_3 : pounds invested in investment C

x_4 : pounds invested in investment D

x_5 : pounds invested in investment E

s_t : pounds invested in money market funds at time t , $t \in \{0, 1, 2\}$

■ Objective function

$$(1) \quad \max z = x_2 + 1.9x_4 + 1.5x_5 + 1.08s_2$$



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■ Constraints

$$(1) \quad x_1 + x_3 + x_4 + s_0 = 100000 \quad (\textit{investment at time 0})$$

$$(2) \quad 0.5x_1 + 1.2x_3 + 1.08s_0 = x_2 + s_1 \quad (\textit{investment at time 1})$$

$$(3) \quad x_1 + 0.5x_2 + 1.08s_1 = x_5 + s_2 \quad (\textit{investment at time 2})$$

$$(4) \quad x_i \leq 75000, \quad \forall i \in \{1, \dots, 5\} \quad (\textit{maximum single investment})$$

$$(5) \quad x_i \geq 0, \quad \forall i \in \{1, \dots, 5\} \quad (\textit{sign constraint})$$

$$(6) \quad s_t \geq 0, \quad \forall t \in \{0, 1, 2\} \quad (\textit{sign constraint})$$



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From Winston, Wayne L. - Operations Research

- *Sunco manufactures three types of gasoline*
- *Each gasoline type is a blending from three types of crude*
- *Price per barrel of gasoline and crude are given below*
- *At most 5000 barrels of each crude is purchased daily*
- *Octane rating and sulfur level for each gasoline is given below*
- *Octane rating and sulfur level of each crude is given below*
- *It costs £4 to refine each barrel of crude*
- *Refinery capacity is 14000 barrels per day*
- *Demand for gasoline is given below*
- *Gasoline advertisement: increase of 10 barrels per £1 spent*

Gas	Demand	Sells	Octane	Sulfur (%)
1	3000	£70	10	1.0
2	2000	£60	8	2.0
3	1000	£50	6	1.0

Crude	Bought	Octane	Sulfur(%)
1	£45	12	0.5
2	£35	6	2.0
3	£25	8	3.0

Maximize Sunco's profit



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■ Decision variables

x_{ij} : barrels of crude oil i used daily to produce gas j ; $i \in \{1, 2, 3\}$,
 $j \in \{1, 2, 3\}$

a_j : pounds spent daily on advertising gas j , $j \in \{1, 2, 3\}$

■ Objective function

$$\begin{aligned} \max z = & 21x_{11} + 11x_{12} + x_{13} + 31x_{21} + 21x_{22} - 11x_{23} + \\ (1) & + 41x_{31} + 31x_{32} + 21x_{33} - a_1 - a_2 - a_3 \end{aligned}$$



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- (1) $x_{11} + x_{21} + x_{31} = 3000 + 10a_1$ (*gas 1 daily demand*)
- (2) $x_{12} + x_{22} + x_{32} = 2000 + 10a_2$ (*gas 2 daily demand*)
- (3) $x_{13} + x_{23} + x_{33} = 1000 + 10a_3$ (*gas 3 daily demand*)
- (4) $x_{11} + x_{12} + x_{13} \leq 5000$ (*crude 1 max daily purchase*)
- (5) $x_{21} + x_{22} + x_{23} \leq 5000$ (*crude 2 max daily purchase*)
- (6) $x_{31} + x_{32} + x_{33} \leq 5000$ (*crude 3 max daily purchase*)
- (7) $\sum_{ij} x_{ij} \leq 14000, \forall i \in \{1, 2, 3\}, \forall j \in \{1, 2, 3\}$ (*refinery capacity*)
- (8) $2x_{11} - 4x_{21} - 2x_{31} \geq 0$ (*gas 1 octane level*)
- (9) $4x_{12} - 2x_{22} \geq 0$ (*gas 2 octane level*)
- (10) $6x_{13} + 2x_{33} \geq 0$ (*gas 3 octane level : redundant*)
- (11) $-0.005x_{11} + 0.01x_{21} + 0.02x_{31} \leq 0$ (*gas 1 sulfur level*)
- (12) $-0.015x_{12} + 0.01x_{32} \leq 0$ (*gas 2 sulfur level*)
- (13) $-0.005x_{13} + 0.01x_{23} + 0.02x_{33} \leq 0$ (*gas 3 sulfur level*)



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■ Show me the results!



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- Discussion lists
 - ◆ Main discussion list: help, development, porting, enhancement request
 - help-glpk@gnu.org



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 - help-glpk@gnu.org
 - ◆ Bugs
 - bug-glpk@gnu.org



Maintainer

GLPK is maintained by Andrew Makhorin (mao@mai2.rcnet.ru)

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- Open Source Community
- Linuxconf Europe 2007 organizers

Thank you!



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